Algebra 2 Honors – Summer Assignment

For each lesson, study the vocabulary and the examples. Then complete the exercises on separate looseleaf. (The exercises appear after the examples.)

- Lesson 1.1 Study Guide: Exercises 1-11
- Lesson 1.2 Study Guide: Exercises 1-10
- Lesson 1.3 Study Guide: Exercises 1-9
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Show all work for each problem where it is appropriate. Try not to use a calculator, unless you are told to do so in the directions. This assignment is due on the first day of classes.

Name _____ Date _____

LESSON 1.1

Study Guide

For use with pages 2–9

GOAL **Study properties of real numbers.**

Vocabulary

The **opposite,** or additive inverse, of any number b is $-b$. If b is positive, then $-b$ is negative. If b is negative, then $-b$ is positive.

The **reciprocal,** or multiplicative inverse, of any nonzero number b is $-\frac{1}{b}$.

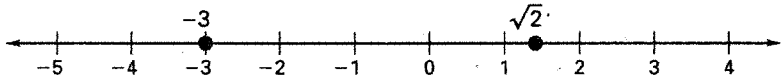
EXAMPLE 1 Graph real numbers on a number line

Graph the real numbers -3 and $\sqrt{2}$ on a number line.

Solution

Use a calculator to approximate $\sqrt{2}$ to the nearest tenth: $\sqrt{2} \approx 1.4$.

On the number line, graph $\sqrt{2}$ between 1 and 2 as shown below.



Exercises for Example 1

Graph the numbers on a number line.

1. $\sqrt{3}$ and -1
2. -4 and 5
3. $\frac{1}{2}$ and 0

Graph the numbers on a number line. Order the numbers in increasing order.

4. $2, -3, \frac{2}{3}, -2$, and $\sqrt{6}$
5. $4, -5, \frac{1}{4}, 0$, and -4

EXAMPLE 2 Identify properties of real numbers

Identify the property that the statement illustrates.

a. $3 \cdot 4 = 4 \cdot 3$

b. $-7 + 7 = 0$

c. $0 + 8 = 8$

d. $5(4 + 2) = 5(4) + 5(2)$

Solution

a. Commutative property of multiplication

b. Inverse property of addition

c. Identity property of addition

d. Distributive property

LESSON 1.1 **Study Guide** *continued*
For use with pages 2–9

Exercises for Example 2

Identify the property that the statement illustrates.

6. $36 + (12 + 11) = (36 + 12) + 11$

7. $22 \cdot \frac{1}{22} = 1$

8. $20(33 + 40) = 20(33) + 20(40)$

LESSON 1.1

EXAMPLE 3 Use properties and definitions of operations

Use properties and definitions of operations to show that $(c \div 2)12 = 6c$. Justify each step.

Solution

$(c \div 2)12 = \left(c \cdot \frac{1}{2}\right)12$ Definition of division

$= c \cdot \left(\frac{1}{2} \cdot 12\right)$ Associative property of multiplication

$= c \cdot 6$ Simplify.

$= 6 \cdot c$ Commutative property of multiplication

Exercises for Example 3

Use properties and definitions of operations to show that the statement is true. Justify each step.

9. $6 - 6 + y = y$

10. $2a + 7 + 3a = 7 + 5a$

EXAMPLE 4 Use unit analysis with operations

You work for 3 hours at $4.60 per hour. How much do you earn?

Solution

$(3 \text{ hours}) \left(\frac{4.60 \text{ dollars}}{1 \text{ hour}}\right) = \$13.80$

Exercise for Example 4

11. You travel for 5.5 hours at 60 miles per hour. How far do you go?

Name _____ Date _____

LESSON 1.2 Study Guide

For use with pages 10–17

GOAL **Evaluate and simplify expressions involving real numbers.**

Vocabulary

An expression formed by repeated multiplication of the same factor is called a **power.**

A **variable** is a letter that is used to represent one or more numbers.

In an expression that can be written as a sum, the parts added together are called the **terms.**

When a term is a product of a number and a power of a variable, the number is called the **coefficient** of the power.

A statement that equates two equivalent expressions is called an **identity.**

EXAMPLE 1 Evaluate powers

a. -3^2 **b.** $(-3)^2$

Solution

a. $-(3 \cdot 3) = -9$ **b.** $(-3)(-3) = 9$

Order of Operations

First, do operations that occur within grouping symbols.

Next, evaluate powers.

Then, do multiplications and divisions from left to right.

Finally, do additions and subtractions from left to right.

EXAMPLE 2 Evaluate an algebraic expression

Evaluate $3x^2 + 4x - 5$ when $x = -2$.

Solution

$3x^2 + 4x - 5 = 3(-2)^2 + 4(-2) - 5$	Substitute -2 for x.
$= 3(4) + 4(-2) - 5$	Evaluate power.
$= 12 - 8 - 5$	Multiply.
$= -1$	Subtract.

Exercises for Examples 1 and 2

1. Evaluate $-x^2 + 3x$ when $x = 2$.

2. Evaluate $2x^2 - x + 1$ when $x = -1$.

3. Evaluate $6 - b^2$ when $b = 5$.

4. Evaluate $3d^2 + 4d$ when $d = -2$.

Name _____ Date _____

LESSON 1.2 **Study Guide** *continued*
For use with pages 10–17

EXAMPLE 3 Simplify by combining like terms

$-6(y - 2) + 4(y - 1) = -6y + 12 + 4y - 4$ Distributive property

$= (-6y + 4y) + (12 - 4)$ Group like terms.

$= -2y + 8$ Combine like terms.

Exercises for Example 3

Simplify by combining like terms.

5. $3t + 5t^2 - 2t + 6t^2$

6. $7(q - 2) + 5q + 14$

7. $-4(m - 2) + 3(m + 1)$

8. $8d + 2d^2 - 3(d + d^2)$

EXAMPLE 4 Simplify a mathematical model

The width of a rectangular patio is $n + 3$ and the length of the patio is $7n$. Write and simplify an expression that represents the perimeter of the patio. Then find the perimeter if $n = 2$ feet.

Solution

Write a verbal model. Then write an algebraic expression.

Number of sides	·	Width (in feet)	+	Number of sides	·	Length (in feet)
↓		↓		↓		↓
2	·	$(n + 3)$	+	2	·	$7n$

An algebraic expression for the perimeter is $P = 2(n + 3) + 2(7n)$.

$2(n + 3) + 2(7n) = 2n + 6 + 14n$ Distributive property

$= (2n + 14n) + 6$ Group like terms.

$= 16n + 6$ Combine like terms.

When $n = 2$, the perimeter is $16(2) + 6 = 32 + 6 = 38$ feet.

Exercises for Example 4

9. The width of a rectangular patio is $2n + 3$ and the length of the patio is $5n$. Write and simplify an expression that represents the perimeter of the patio. Then find the perimeter if $n = 4$ feet.

10. You purchase 12 packages of wrapping paper. Large packages cost \$7 and small packages cost \$4.50. Write and simplify an expression that represents the total cost if n of the 12 packages are large packages. Then find the total cost if 8 of the 12 packages are large packages.

Name _____ Date _____

LESSON 1.3 Study Guide

For use with pages 18–25

GOAL **Solve linear equations.**

Vocabulary

An **equation** is a statement that two expressions are equal.

A **linear equation** in one variable is an equation that can be written in the form $ax + b = 0$ where a and b are constants and $a \neq 0$.

A number is a **solution** of an equation if substituting the number for the variable results in a true statement.

Two equations are **equivalent equations** if they have the same solution(s).

EXAMPLE 1 Solve an equation with a variable on one side

Solve $6x - 8 = 10$.

Solution

$6x - 8 = 10$	Write original equation.
$6x = 18$	Add 8 to each side.
$x = 3$	Divide each side by 6.

EXAMPLE 2 Solve an equation with a variable on both sides

Solve $8z + 7 = -2z - 3$.

Solution

$8z + 7 = -2z - 3$	Write original equation.
$10z + 7 = -3$	Add $2z$ to each side.
$10z = -10$	Subtract 7 from each side.
$z = -1$	Divide each side by 10.

Exercises for Examples 1 and 2

Solve the equation. Check your solution.

1. $14x = 7$ **2.** $3n + 2 = 14$ **3.** $-6t - 5 = 13$

4. $11q - 4 = 6q - 9$ **5.** $5a - 1 = 2a + 11$ **6.** $-2m + 3 = 7m - 6$

Name _____ Date _____

LESSON 1.3

Study Guide *continued*

For use with pages 18–25

EXAMPLE 3 Solve an equation using the distributive property

Solve $2(3x + 1) = -3(x - 2)$.

Solution

$2(3x + 1) = -3(x - 2)$	Write original equation.
$6x + 2 = -3x + 6$	Distributive property
$9x + 2 = 6$	Add $3x$ to each side.
$9x = 4$	Subtract 2 from each side.
$x = \frac{4}{9}$	Divide each side by 9.

Exercises for Example 3

7. Solve $4(2x - 1) = 3(x + 2)$.

8. Solve $5(x + 3) = -(x - 3)$.

EXAMPLE 4 Solve a work problem

It takes you 3 hours to mow a lawn and it takes your sister 2 hours to mow a lawn. How long does it take the two of you to mow 5 lawns if you work together?

Solution

Solve the equation $\frac{1}{3}t + \frac{1}{2}t = 5$ for t.

$\frac{1}{3}t + \frac{1}{2}t = 5$	Write equation.
$6\left(\frac{1}{3}t + \frac{1}{2}t\right) = 6(5)$	Multiply each side by the LCD, 6.
$2t + 3t = 30$	Distributive property
$5t = 30$	Combine like terms.
$t = 6$	Divide each side by 5.

It will take 6 hours to mow 5 lawns if you work together.

Exercise for Example 4

9. Rework Example 4 to find how long it takes the two of you to mow 12 lawns if you work together.

Name _____ Date _____

LESSON 1.4

LESSON 1.4 Study Guide

For use with pages 26–32

GOAL **Rewrite and evaluate formulas and equations.**

Vocabulary

A **formula** is an equation that relates two or more quantities, usually represented by variables.

To **solve for a variable** means to rewrite an equation as an equivalent equation in which the variable is on one side and does not appear on the other side.

EXAMPLE 1 Rewrite a formula with two variables

Solve the formula $F = \frac{9}{5}C + 32$ for C.

$F = \frac{9}{5}C + 32$	Write temperature formula.
$5F = 9C + 160$	Multiply each side by 5.
$5F - 160 = 9C$	Subtract 160 from each side.
$\frac{5F - 160}{9} = C$	Divide each side by 9.

EXAMPLE 2 Rewrite a formula with three variables

Solve the formula $A = \ell w$ for ℓ. Then find ℓ when $A = 5$ square centimeters and $w = 2$ centimeters.

STEP 1 Solve the formula for ℓ.

$A = \ell w$	Write area formula.
$\frac{A}{w} = \frac{\ell w}{w}$	Divide each side by w.
$\frac{A}{w} = \ell$	Simplify.

STEP 2 Substitute the given values into the rewritten formula.

$\ell = \frac{5}{2} = 2.5$	Substitute 5 for A and 2 for w.

The length of the rectangle is 2.5 centimeters.

Exercises for Examples 1 and 2

1. Solve the formula $C = 2\pi r$ for r. Then find the radius of a circle with a circumference of 88 inches.
2. Solve the formula $P = 2\ell + 2w$ for w. Then find the width of a rectangle with a length of 11.5 centimeters and a perimeter of 92 centimeters.

Name _____ Date _____

LESSON 1.4 **Study Guide** *continued*
For use with pages 26–32

EXAMPLE 3 Rewrite a nonlinear equation

Solve the equation $xy - y = 9$ for y. Then find the value of y for $x = 4$.

STEP 1 Solve the equation for y.

$xy - y = 9$ Write original equation.

$y(x - 1) = 9$ Distributive property

$y = \frac{9}{x - 1}$ Divide each side by $x - 1$.

STEP 2 Substitute the given value into the rewritten equation.

$y = \frac{9}{4 - 1} = 3$ Substitute 4 for x and simplify.

Exercises for Example 3

Solve the equation for y. Find the value of y for the given value of x.

3. $xy + y = 8; x = 3$ **4.** $2xy + y = 6; x = 1$ **5.** $4xy - y = 7; x = 2$

EXAMPLE 4 Solve a multi-step problem

You buy x shirts for \$8 each and y hats for \$4 each. Write an equation that represents your total purchases T (in dollars) and solve the equation for y. Evaluate the equation for $T = 28$ and $x = 2$.

STEP 1 Write a verbal model. Then write an equation.

Total purchases	=	Prices of shirts	•	Number of shirts	+	Price of hats	•	Number of hats

An equation is $T = 8x + 4y$.

STEP 2 Solve the equation for y.

$T = 8x + 4y$ Write equation.

$T - 8x = 4y$ Subtract $8x$ from each side.

$\frac{T - 8x}{4} = y$ Divide each side by 4.

STEP 3 Calculate y. If $T = 28$ and $x = 2$, then $y = \frac{28 - 8(2)}{4} = 3$.

If 2 shirts are purchased, then 3 hats are purchased.

Exercise for Example 4

6. In Example 4, how many hats can you buy if you buy 1 shirt and spend \$16?

Name _____ Date _____

LESSON 1.5 Study Guide

For use with pages 34–40

GOAL **Solve problems using verbal models.**

Vocabulary

When solving real-life problems, write an equation called a **verbal model** in words before you write the equation in mathematical symbols.

EXAMPLE 1 Use a formula

A rectangular flower garden has an area of 125 square feet. If the length of the garden is 12 feet, what is the width of the garden?

Solution

You can use the formula for the area of a rectangle as a verbal model.

Area (square feet)	=	Length (feet)	•	Width (feet)
↓		↓		↓
125	=	12	•	w

An equation for this situation is $125 = 12 \cdot w$. Solve for w.

$125 = 12 \cdot w$ Write equation.

$10.4 \approx w$ Divide each side by 12.

The width of the garden is about 10.4 feet.

Exercises for Example 1

1. The perimeter of a rectangular city park is 1080 yards. The width of the park is 240 yards. What is the length of the park?
2. A train travels at a speed of 44 miles per hour. How long will it take the train to travel 154 miles?

EXAMPLE 2 Look for a pattern

Look for a pattern in the table. Then write an equation that represents the table.

x	0	1	2	3
y	0	3	6	9

Solution

The x-values increase by 1 and the y-values increase by 3. You can use this pattern to write an equation $y = 3x$.

Name _____ Date _____

LESSON 1.5 **Study Guide** *continued*
For use with pages 34–40

Exercises for Example 2

Look for a pattern in the table. Then write an equation that represents the table.

3.

x	0	1	2	3
y	0	−2	−4	−6

4.

x	0	1	2	3
y	1	5	9	13

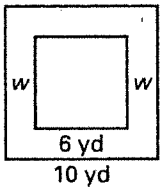
LESSON 1.5

EXAMPLE 3 Draw a diagram

You are designing a square flower garden surrounded by a brick sidewalk of uniform width. The garden has a side length of 6 yards. The side length of the outside square is 10 yards. Draw a diagram to find the width of the sidewalk.

Solution

Begin by drawing and labeling a diagram, as shown below.



From the diagram, you can write and solve an equation to find w.

$w + 6 + w = 10$ Write equation.

$2w + 6 = 10$ Combine like terms.

$2w = 4$ Subtract 6 from each side.

$w = 2$ Divide each side by 2.

The width of the sidewalk is 2 yards.

Exercises for Example 3

5. You want to create an open rectangular box from a rectangular piece of cardboard. The cardboard has a length of 14 inches and you will cut 2 inch squares from each corner. Draw a diagram to find the length of the box.

6. A piece of fabric is 52 inches long. You cut the fabric into two pieces. The first piece is x inches long. The second piece is 14 inches longer than the first piece. Draw and label a diagram of the fabric. Then write and solve an equation to find x.

7. You want to create an open rectangular box from a square piece of cardboard. The cardboard is 20 inches by 20 inches and you will cut 3 inch squares from each corner. Draw a diagram to find the length of the box.

Name _____ Date _____

LESSON 1.6 Study Guide

For use with pages 41–47

GOAL **Solve linear inequalities.**

Vocabulary

A **linear inequality** in one variable can be written in one of the following forms, where a and b are real numbers and $a \neq 0$:
$ax + b < 0$, $ax + b > 0$, $ax + b \leq 0$, $ax + b \geq 0$

A **compound inequality** consists of two simple inequalities joined by "and" or "or".

EXAMPLE 1 Solve an inequality with a variable on one side

You have $18 to spend at a movie theater. You spend $8 for admission. You want to buy soft drinks that cost $2.50 each. Describe the possible numbers of soft drinks that you can buy.

Solution

STEP 1 **Write** a verbal model. Then write an inequality.

Admission fee	+	Cost per drink	·	Number of drinks	≤	Amount you can spend
↓		↓		↓		↓
8	+	2.50	·	d	≤	18

An inequality is $8 + 2.5d \leq 18$.

STEP 2 **Solve** the inequality.

$8 + 2.5d \leq 18$ Write inequality.

$2.5d \leq 10$ Subtract 8 from each side.

$d \leq 4$ Divide each side by 2.5.

You can buy 4 drinks or fewer.

Exercises for Example 1

1. Rework Example 1 if you have $16 to spend, soft drinks cost $1.50, and admission is $10.

2. You have $25 to park your automobile in a parking garage. The charge for the first hour is $4 and the charge for each additional hour is $3. Describe the possible number of hours that you can park.

Name _____ Date _____

LESSON 1.6

Study Guide *continued*

For use with pages 41–47

EXAMPLE 2 Solve an inequality with a variable on both sides

Solve $-4x + 3 < x - 2$. Then graph the solution.

$-4x + 3 < x - 2$	Write original inequality.
$-5x < -5$	Subtract x and 3 from each side.
$x > 1$	Divide each side by -5 and reverse the inequality.

EXAMPLE 3 Solve an "and" compound inequality

Solve $-3 \leq 2x + 5 < 7$. Then graph the solution.

$-3 \leq 2x + 5 < 7$	Write original inequality.
$-3 - 5 \leq 2x + 5 - 5 < 7 - 5$	Subtract 5 from each expression.
$-8 \leq 2x < 2$	Simplify.
$-4 \leq x < 1$	Divide each expression by 2.

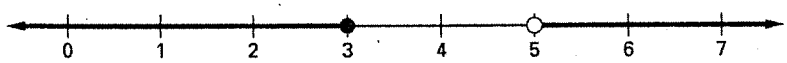
EXAMPLE 4 Solve an "or" compound inequality

Solve $3x + 1 \leq 10$ or $4x - 1 > 19$. Then graph the solution.

A solution of this inequality is a solution of either of its parts.

First Inequality		Second Inequality	
$3x + 1 \leq 10$	Write first inequality.	$4x - 1 > 19$	Write second inequality.
$3x \leq 9$	Subtract 1 from each side.	$4x > 20$	Add 1 to each side.
$x \leq 3$	Divide each side by 3.	$x > 5$	Divide each side by 4.

The graph is shown below. The solution is all real numbers less than or equal to 3 or greater than 5.



Exercises for Examples 2, 3, and 4

Solve the inequality. Then graph the solution.

3. $7x - 12 \leq -x + 4$

4. $-8x + 9 > 2x - 1$

5. $0 < 3x - 6 \leq 3$

6. $-5 \leq 2x + 5 \leq -1$

7. $x < 3$ or $x + 1 > 7$

8. $2x + 3 < 7$ or $x - 2 \geq 4$

Name _____ Date _____

LESSON 1.7

Study Guide

For use with pages 50–58

GOAL **Solve absolute value equations and inequalities.**

Vocabulary

The **absolute value** of a number x, written $|x|$, is the distance the number is from 0 on a number line.

An **extraneous solution** is an apparent solution that must be rejected because it does not satisfy the original equation.

EXAMPLE 1 Solve an absolute value equation

Solve $|8 - 4x| = 12$.

$\|8 - 4x\| = 12$			Write original equation.
$8 - 4x = 12$	or	$8 - 4x = -12$	Expression can equal 12 or -12.
$-4x = 4$	or	$-4x = -20$	Subtract 8 from each side.
$x = -1$	or	$x = 5$	Divide each side by -4.

The solutions are -1 and 5.

Exercises for Example 1

Solve the equation.

1. $|x + 3| = 7$ **2.** $|x - 2| = 6$ **3.** $|2x - 5| = 9$ **4.** $|2x + 1| = 9$

EXAMPLE 2 Check for extraneous solutions

Solve $|2x + 6| = 4x$. Check for extraneous solutions.

$\|2x + 6\| = 4x$			Write original equation.
$2x + 6 = 4x$	or	$2x + 6 = -4x$	Expression can equal $4x$ or $-4x$.
$6 = 2x$	or	$6 = -6x$	Subtract $2x$ from each side.
$3 = x$	or	$-1 = x$	Solve for x.

Check the apparent solutions to see if either is extraneous.

$$|2x + 6| = 4x \qquad |2x + 6| = 4x$$
$$|2(3) + 6| \stackrel{?}{=} 4(3) \qquad |2(-1) + 6| \stackrel{?}{=} 4(-1)$$
$$|12| \stackrel{?}{=} 12 \qquad |4| \stackrel{?}{=} -4$$
$$12 = 12 \qquad 4 \neq -4$$

The solution is 3. Reject -1 because it is an extraneous solution.

LESSON 1.7

Name _____ Date _____

LESSON 1.7

Study Guide *continued*

For use with pages 50–58

Exercises for Example 2

Solve the equation. Check for extraneous solutions.

5. $|x + 2| = 3x$ **6.** $|3x + 3| = 6x$ **7.** $|2x + 4| = 6x$

EXAMPLE 3 Solve an inequality of the form $|ax + b| > c$

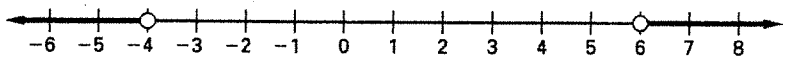
Solve $|x - 1| > 5$. Then graph the solution.

Solution

The absolute value equation is equivalent to $x - 1 < -5$ or $x - 1 > 5$.

First Inequality		Second Inequality
$x - 1 < -5$	Write inequalities.	$x - 1 > 5$
$x < -4$	Add 1 to each side.	$x > 6$

The solutions are all real numbers less than -4 or greater than 6. The graph is shown below.



EXAMPLE 4 Solve an inequality of the form $|ax + b| \leq c$

Solve $|x - 3| \leq 5$. Then graph the solution.

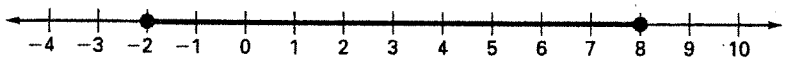
Solution

$|x - 3| \leq 5$ Write original inequality.

$-5 \leq x - 3 \leq 5$ Write equivalent compound inequality.

$-2 \leq x \leq 8$ Add 3 to each expression.

The solution is all real numbers greater than or equal to -2 and less than or equal to 8. The graph is shown below.



Exercises for Examples 3 and 4

Solve the inequality. Then graph the solution.

8. $|x - 3| > 5$ **9.** $|x + 7| > 2$ **10.** $|2x + 1| \geq 5$

11. $|x - 6| \leq 4$ **12.** $|x + 7| < 2$ **13.** $|x + 1| \leq 5$