MHS **148/xx: PRE-CALCULUS HONORS** **SUNG**

2021-2022

SUMMER ASSIGNMENT

You are receiving this Summer Assignment now so that you can plan ahead and work on this assignment on a schedule of your own choosing that fits your plans for the summer. The focus of this assignment is on factoring polynomials and solving (mostly) quadratic equations. Attached are pages from our Pre-Calculus Honors textbook. On loose-leaf, do the following problems:

- Section A.3, pp. A33-A34: Ex. 37-79 odd

- Section A.5, pp. A56-A57: Ex. 25-75 odd

For each problem, show **all** work where appropriate. Try to avoid using a calculator for any of these problems. Check your answers (answer key is also attached) using a pen.

Bring the loose-leaf with all of your work with you on the first day that our class meets (this may be during Orientation). There will be a grade for this assignment. If you submit just a list of answers with little or no work, you will not receive credit. If there are any questions, email Mr. Sung (esung@mercyhighschool.com).

A.3 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. For the polynomial $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$, $a_n \neq 0$, the degree is _____, the leading coefficient is _____, and the constant term is _____.
2. A polynomial with one term is a _____, while a polynomial with two terms is a _____ and a polynomial with three terms is a _____.
3. To add or subtract polynomials, add or subtract the _____ _____ by adding or subtracting their coefficients.
4. The letters in "FOIL" stand for F _____, O _____, I _____, and L _____.
5. The process of writing a polynomial as a product is called _____.
6. A polynomial is _____ _____ when each of its factors is prime.
7. A _____ _____ _____ is the square of a binomial, and it has the form $u^2 + 2uv + v^2$ or $u^2 - 2uv + v^2$.
8. Sometimes, polynomials with more than three terms can be factored by _____.

Skills and Applications

Writing Polynomials in Standard Form

In Exercises 9–14, (a) write the polynomial in standard form, (b) identify the degree and leading coefficient of the polynomial, and (c) state whether the polynomial is a monomial, a binomial, or a trinomial.

9. $7x$

10. 3

11. $14x - \frac{1}{2}x^5$

12. $3 + 2x$

13. $1 + 6x^4 - 4x^5$

14. $-y + 25y^2 + 1$

Adding or Subtracting Polynomials

In Exercises 15–18, add or subtract and write the result in standard form.

15. $(6x + 5) - (8x + 15)$

16. $(2x^2 + 1) - (x^2 - 2x + 1)$

17. $(15x^2 - 6) + (-8.3x^3 - 14.7x^2 - 17)$

18. $(15.6w^4 - 14w - 17.4) + (16.9w^4 - 9.2w + 13)$

Multiplying Polynomials

In Exercises 19–36, multiply the polynomials.

19. $3x(x^2 - 2x + 1)$

20. $y^2(4y^2 + 2y - 3)$

21. $-5z(3z - 1)$

22. $-3x(5x + 2)$

23. $(3x - 5)(2x + 1)$

24. $(7x - 2)(4x - 3)$

25. $(x^2 - x + 2)(x^2 + x + 1)$

26. $(2x^2 - x + 4)(x^2 + 3x + 2)$

27. $(x + 10)(x - 10)$

28. $(4a + 5b)(4a - 5b)$

29. $(2x + 3)^2$

30. $(8x + 3)^2$

31. $(x + 3)^3$

32. $(3x + 2y)^3$

33. $[(x - 3) + y]^2$

34. $[(x + 1) - y]^2$

35. $[(m - 3) + n][(m - 3) - n]$

36. $[(x - 3y) + z][(x - 3y) - z]$

Factoring Out a Common Factor

In Exercises 37–40, factor out the common factor.

37. $2x^3 - 6x$

38. $3z^3 - 6z^2 + 9z$

39. $3x(x - 5) + 8(x - 5)$

40. $(x + 3)^2 - 4(x + 3)$

Factoring the Difference of Two Squares

In Exercises 41–44, completely factor the difference of two squares.

41. $25y^2 - 4$

42. $81 - 36z^2$

43. $(x - 1)^2 - 4$

44. $25 - (z + 5)^2$

Factoring a Perfect Square Trinomial

In Exercises 45–50, factor the perfect square trinomial.

45. $x^2 - 4x + 4$

46. $4t^2 + 4t + 1$

47. $25z^2 - 30z + 9$

48. $36y^2 + 84y + 49$

49. $4y^2 - 12y + 9$

50. $9u^2 + 24uv + 16v^2$

Factoring the Sum or Difference of Two Cubes

In Exercises 51–54, factor the sum or difference of two cubes.

51. $x^3 + 125$

52. $x^3 - 8$

53. $8t^3 - 1$

54. $27t^3 + 8$

Factoring a Trinomial **In Exercises 55–62, factor the trinomial.**

55. $x^2 + x - 2$
56. $s^2 - 5s + 6$
57. $3x^2 + 10x - 8$
58. $2x^2 - 3x - 27$
59. $5x^2 + 31x + 6$
60. $8x^2 + 51x + 18$
61. $-5y^2 - 8y + 4$
62. $-6z^2 + 17z + 3$

Factoring by Grouping **In Exercises 63–68, factor by grouping.**

63. $x^3 - x^2 + 2x - 2$
64. $x^3 + 5x^2 - 5x - 25$
65. $2x^3 - x^2 - 6x + 3$
66. $3x^3 + x^2 - 15x - 5$
67. $3x^5 + 6x^3 - 2x^2 - 4$
68. $8x^5 - 6x^2 + 12x^3 - 9$

Factoring a Trinomial by Grouping **In Exercises 69–72, factor the trinomial by grouping.**

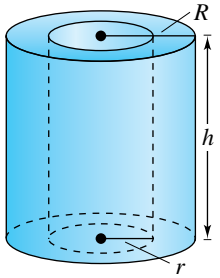
69. $2x^2 + 9x + 9$
70. $6x^2 + x - 2$
71. $6x^2 - x - 15$
72. $12x^2 - 13x + 1$

Factoring Completely **In Exercises 73–82, completely factor the expression.**

73. $6x^2 - 54$
74. $12x^2 - 48$
75. $x^3 - x^2$
76. $x^3 - 16x$
77. $2x^2 + 4x - 2x^3$
78. $9x^2 + 12x - 3x^3$
79. $5 - x + 5x^2 - x^3$
80. $3u - 2u^2 + 6 - u^3$
81. $2(x - 2)(x + 1)^2 - 3(x - 2)^2(x + 1)$
82. $2(x + 1)(x - 3)^2 - 3(x + 1)^2(x - 3)$

83. Geometry The cylindrical shell shown in the figure has a volume of

$V = \pi R^2 h - \pi r^2 h.$



(a) Factor the expression for the volume.

(b) From the result of part (a), show that the volume is 2π(average radius)(thickness of the shell)h.

84. Chemistry The rate of change of an autocatalytic chemical reaction is $kQx - kx^2$, where Q is the amount of the original substance, x is the amount of substance formed, and k is a constant of proportionality. Factor the expression.

Suwit Ngaokaew/Shutterstock.com

Exploration

True or False? **In Exercises 85–87, determine whether the statement is true or false. Justify your answer.**

85. The product of two binomials is always a second-degree polynomial.

86. The sum of two binomials is always a binomial.

87. The difference of two perfect squares can be factored as the product of conjugate pairs.

88. Error Analysis Describe the error.

$$9x^2 - 9x - 54 = (3x + 6)(3x - 9)$$
$$= 3(x + 2)(x - 3)$$

89. Degree of a Product Find the degree of the product of two polynomials of degrees m and n.

90. Degree of a Sum Find the degree of the sum of two polynomials of degrees m and n, where $m < n$.

91. Think About It When the polynomial

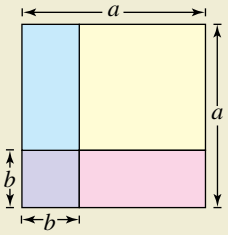
$-x^3 + 3x^2 + 2x - 1$

is subtracted from an unknown polynomial, the difference is $5x^2 + 8$. Find the unknown polynomial.

92. Logical Reasoning Verify that $(x + y)^2$ is not equal to $x^2 + y^2$ by letting $x = 3$ and $y = 4$ and evaluating both expressions. Are there any values of x and y for which $(x + y)^2 = x^2 + y^2$? Explain.

93. Think About It Give an example of a polynomial that is prime.

94. HOW DO YOU SEE IT? The figure shows a large square with an area of a^2 that contains a smaller square with an area of b^2.



(a) Describe the regions that represent $a^2 - b^2$. How can you rearrange these regions to show that $a^2 - b^2 = (a - b)(a + b)$?

(b) How can you use the figure to show that $(a - b)^2 = a^2 - 2ab + b^2$?

(c) Draw another figure to show that $(a + b)^2 = a^2 + 2ab + b^2$. Explain how the figure shows this.

Factoring with Variables in the Exponents **In Exercises 95 and 96, factor the expression as completely as possible.**

95. $x^{2n} - y^{2n}$
96. $x^{3n} + y^{3n}$

A.5 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. An _____ is a statement that equates two algebraic expressions.
2. A linear equation in one variable x is an equation that can be written in the standard form _____.
3. An _____ solution is a solution that does not satisfy the original equation.
4. Four methods for solving quadratic equations are _____, extracting _____ _____, _____ the _____, and the _____ _____.

Skills and Applications

Solving a Linear Equation **In Exercises 5–12, solve the equation and check your solution. (If not possible, explain why.)**

5. $x + 11 = 15$
6. $7 - x = 19$
7. $7 - 2x = 25$
8. $7x + 2 = 23$
9. $3x - 5 = 2x + 7$
10. $4y + 2 - 5y = 7 - 6y$
11. $x - 3(2x + 3) = 8 - 5x$
12. $9x - 10 = 5x + 2(2x - 5)$

Solving a Rational Equation **In Exercises 13–24, solve the equation and check your solution. (If not possible, explain why.)**

13. $\frac{3x}{8} - \frac{4x}{3} = 4$
14. $\frac{5x}{4} + \frac{1}{2} = x - \frac{1}{2}$
15. $\frac{5x - 4}{5x + 4} = \frac{2}{3}$
16. $\frac{10x + 3}{5x + 6} = \frac{1}{2}$
17. $10 - \frac{13}{x} = 4 + \frac{5}{x}$
18. $\frac{1}{x} + \frac{2}{x - 5} = 0$
19. $\frac{x}{x + 4} + \frac{4}{x + 4} + 2 = 0$
20. $\frac{7}{2x + 1} - \frac{8x}{2x - 1} = -4$
21. $\frac{2}{(x - 4)(x - 2)} = \frac{1}{x - 4} + \frac{2}{x - 2}$
22. $\frac{12}{(x - 1)(x + 3)} = \frac{3}{x - 1} + \frac{2}{x + 3}$
23. $\frac{1}{x - 3} + \frac{1}{x + 3} = \frac{10}{x^2 - 9}$
24. $\frac{1}{x - 2} + \frac{3}{x + 3} = \frac{4}{x^2 + x - 6}$

Solving a Quadratic Equation by Factoring **In Exercises 25–34, solve the quadratic equation by factoring.**

25. $6x^2 + 3x = 0$
26. $8x^2 - 2x = 0$
27. $x^2 + 10x + 25 = 0$
28. $x^2 - 2x - 8 = 0$
29. $3 + 5x - 2x^2 = 0$
30. $4x^2 + 12x + 9 = 0$
31. $16x^2 - 9 = 0$
32. $-x^2 + 8x = 12$
33. $\frac{3}{4}x^2 + 8x + 20 = 0$
34. $\frac{1}{8}x^2 - x - 16 = 0$

Extracting Square Roots **In Exercises 35–42, solve the equation by extracting square roots. When a solution is irrational, list both the exact solution *and* its approximation rounded to two decimal places.**

35. $x^2 = 49$
36. $x^2 = 43$
37. $3x^2 = 81$
38. $9x^2 = 36$
39. $(x - 4)^2 = 49$
40. $(x + 9)^2 = 24$
41. $(2x - 1)^2 = 18$
42. $(x - 7)^2 = (x + 3)^2$

Completing the Square **In Exercises 43–50, solve the quadratic equation by completing the square.**

43. $x^2 + 4x - 32 = 0$
44. $x^2 - 2x - 3 = 0$
45. $x^2 + 4x + 2 = 0$
46. $x^2 + 8x + 14 = 0$
47. $6x^2 - 12x = -3$
48. $4x^2 - 4x = 1$
49. $2x^2 + 5x - 8 = 0$
50. $3x^2 - 4x - 7 = 0$

Using the Quadratic Formula **In Exercises 51–64, use the Quadratic Formula to solve the equation.**

51. $2x^2 + x - 1 = 0$
52. $2x^2 - x - 1 = 0$
53. $9x^2 + 30x + 25 = 0$
54. $28x - 49x^2 = 4$
55. $2x^2 - 7x + 1 = 0$
56. $3x + x^2 - 1 = 0$
57. $12x - 9x^2 = -3$
58. $9x^2 - 37 = 6x$
59. $2 + 2x - x^2 = 0$
60. $x^2 + 10 + 8x = 0$
61. $8t = 5 + 2t^2$
62. $25h^2 + 80h = -61$
63. $(y - 5)^2 = 2y$
64. $(z + 6)^2 = -2z$

Choosing a Method **In Exercises 65–72, solve the equation using any convenient method.**

65. $x^2 - 2x - 1 = 0$

66. $14x^2 + 42x = 0$

67. $(x + 2)^2 = 64$

68. $x^2 - 14x + 49 = 0$

69. $x^2 - x - \frac{11}{4} = 0$

70. $x^2 + 3x - \frac{3}{4} = 0$

71. $3x + 4 = 2x^2 - 7$

72. $(x + 1)^2 = x^2$

Solving a Polynomial Equation **In Exercises 73–76, solve the equation. Check your solutions.**

73. $6x^4 - 54x^2 = 0$

74. $5x^3 + 30x^2 + 45x = 0$

75. $x^3 + 2x^2 - 8x = 16$

76. $x^3 - 3x^2 - x = -3$

Solving a Radical Equation **In Exercises 77–84, solve the equation. Check your solutions.**

77. $\sqrt{5x} - 10 = 0$

78. $\sqrt{x + 8} - 5 = 0$

79. $4 + \sqrt[3]{2x - 9} = 0$

80. $\sqrt[3]{12 - x} - 3 = 0$

81. $\sqrt{x + 8} = 2 + x$

82. $2x = \sqrt{-5x + 24} - 3$

83. $\sqrt{x - 3} + 1 = \sqrt{x}$

84. $2\sqrt{x + 1} - \sqrt{2x + 3} = 1$

Solving an Equation Involving a Rational Exponent **In Exercises 85–88, solve the equation. Check your solutions.**

85. $(x - 5)^{3/2} = 8$

86. $(x^2 - x - 22)^{3/2} = 27$

87. $3x(x - 1)^{1/2} + 2(x - 1)^{3/2} = 0$

88. $4x^2(x - 1)^{1/3} + 6x(x - 1)^{4/3} = 0$

Solving an Absolute Value Function **In Exercises 89–92, solve the equation. Check your solutions.**

89. $|2x - 5| = 11$

90. $|3x + 2| = 7$

91. $|x + 1| = x^2 - 5$

92. $|x^2 + 6x| = 3x + 18$

93. Volume of a Billiard Ball A billiard ball has a volume of 5.96 cubic inches. Find the radius of the billiard ball.

94. Length of a Tank The diameter of a cylindrical propane gas tank is 4 feet. The total volume of the tank is 603.2 cubic feet. Find the length of the tank.

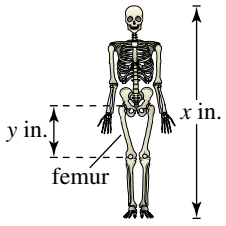
Forensics

In Exercises 95 and 96, use the following information. The relationship between the length of an adult's femur (thigh bone) and the height of the adult can be approximated by the linear equations

$y = 0.514x - 14.75$ Female

$y = 0.532x - 17.03$ Male

where y is the length of the femur in inches and x is the height of the adult in inches (see figure).



95. A crime scene investigator discovers a femur belonging to an adult human female. The bone is 18 inches long. Estimate the height of the female.

96. Officials search a forest for a missing man who is 6 feet 2 inches tall. They find an adult male femur that is 23 inches long. Is it possible that the femur belongs to the missing man?

Exploration

True or False? **In Exercises 97–99, determine whether the statement is true or false. Justify your answer.**

97. An equation can never have more than one extraneous solution.

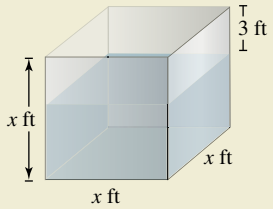
98. The equation $2(x - 3) + 1 = 2x - 5$ has no solution.

99. The equation

$$\sqrt{x + 10} - \sqrt{x - 10} = 0$$

has no solution.

100. HOW DO YOU SEE IT? The figure shows a glass cube partially filled with water.



(a) What does the expression $x^2(x - 3)$ represent?

(b) Given $x^2(x - 3) = 320$, explain how to find the capacity of the cube.

Appendix A

Appendix A.3 *(page A33)*

1. n; a_n; a_0 **3.** like terms **5.** factoring
7. perfect square trinomial
9. (a) $7x$ (b) Degree: 1; Leading coefficient: 7
(c) Monomial
11. (a) $-\frac{1}{2}x^5 + 14x$ (b) Degree: 5; Leading coefficient: $-\frac{1}{2}$
(c) Binomial
13. (a) $-4x^5 + 6x^4 + 1$
(b) Degree: 5; Leading coefficient: -4 (c) Trinomial
15. $-2x - 10$ **17.** $-8.3x^3 + 0.3x^2 - 23$
19. $3x^3 - 6x^2 + 3x$ **21.** $-15z^2 + 5z$ **23.** $6x^2 - 7x - 5$
25. $x^4 + 2x^2 + x + 2$ **27.** $x^2 - 100$ **29.** $4x^2 + 12x + 9$
31. $x^3 + 9x^2 + 27x + 27$ **33.** $x^2 + 2xy + y^2 - 6x - 6y + 9$
35. $m^2 - n^2 - 6m + 9$ **37.** $2x(x^2 - 3)$
39. $(x - 5)(3x + 8)$ **41.** $(5y - 2)(5y + 2)$
43. $(x + 1)(x - 3)$ **45.** $(x - 2)^2$ **47.** $(5z - 3)^2$
49. $(2y - 3)^2$ **51.** $(x + 5)(x^2 - 5x + 25)$
53. $(2t - 1)(4t^2 + 2t + 1)$ **55.** $(x + 2)(x - 1)$
57. $(3x - 2)(x + 4)$ **59.** $(5x + 1)(x + 6)$
61. $-(5y - 2)(y + 2)$ **63.** $(x - 1)(x^2 + 2)$
65. $(2x - 1)(x^2 - 3)$ **67.** $(3x^3 - 2)(x^2 + 2)$
69. $(x + 3)(2x + 3)$ **71.** $(2x + 3)(3x - 5)$
73. $6(x + 3)(x - 3)$ **75.** $x^2(x - 1)$ **77.** $-2x(x + 1)(x - 2)$
79. $(5 - x)(1 + x^2)$ **81.** $-(x - 2)(x + 1)(x - 8)$

83. (a) $\pi h(R + r)(R - r)$ (b) $V = 2\pi\left[\left(\frac{R + r}{2}\right)(R - r)\right]h$
85. False. $(4x^2 + 1)(3x + 1) = 12x^3 + 4x^2 + 3x + 1$
87. True. $a^2 - b^2 = (a + b)(a - b)$ **89.** $m + n$
91. $-x^3 + 8x^2 + 2x + 7$
93. Answers will vary. *Sample answer:* $x^2 - 3$
95. $(x^n + y^n)(x^n - y^n)$

Appendix A.5 *(page A56)*

1. equation **3.** extraneous **5.** 4 **7.** -9 **9.** 12
11. No solution **13.** $-\frac{96}{23}$ **15.** 4 **17.** 3
19. No solution. The variable is divided out.
21. No solution. The solution is extraneous.
23. 5 **25.** $0, -\frac{1}{2}$ **27.** -5 **29.** $-\frac{1}{2}, 3$ **31.** $\pm\frac{3}{4}$
33. $-\frac{20}{3}, -4$ **35.** ± 7 **37.** $\pm 3\sqrt{3} \approx 5.20$ **39.** $-3, 11$
41. $\frac{1 \pm 3\sqrt{2}}{2} \approx 2.62, -1.62$ **43.** $4, -8$ **45.** $-2 \pm \sqrt{2}$
47. $-1 \pm \frac{\sqrt{2}}{2}$ **49.** $\frac{-5 \pm \sqrt{89}}{4}$ **51.** $\frac{1}{2}, -1$ **53.** $-\frac{5}{3}$
55. $\frac{7}{4} \pm \frac{\sqrt{41}}{4}$ **57.** $\frac{2}{3} \pm \frac{\sqrt{7}}{3}$ **59.** $1 \pm \sqrt{3}$ **61.** $2 \pm \frac{\sqrt{6}}{2}$
63. $6 \pm \sqrt{11}$ **65.** $1 \pm \sqrt{2}$ **67.** $-10, 6$ **69.** $\frac{1}{2} \pm \sqrt{3}$
71. $\frac{3}{4} \pm \frac{\sqrt{97}}{4}$ **73.** $0, \pm 3$ **75.** $-2, \pm 2\sqrt{2}$ **77.** 20
79. $-\frac{55}{2}$ **81.** 1 **83.** 4 **85.** 9 **87.** 1 **89.** $8, -3$
91. $-\frac{1}{2} - \frac{\sqrt{17}}{2}, 3$ **93.** $\sqrt[3]{\frac{4.47}{\pi}} \approx 1.12$ in. **95.** 63.7 in.
97. False. See Example 14 on page A54.
99. True. There is no value that satisfies this equation.