

Mr. Jonathan Frank
Mercy High School
Geometry
Summer Preparatory Work

Dear Student,

As you prepare for your studies at Mercy High School in this upcoming school year, your Summer work has been selected for you to learn, practice, or strengthen your skills in three areas critical to correctly understanding and answering problems in Geometry:

- evaluating operations with **fractions**
- correctly using **powers** and **square roots**
- applying and solving **formulas**

An answer key is provided with this packet. Your goal is to **work out** all the problems so that you get the correct answer. You should **complete all of the problems, showing work** (except for the Powers and Square Roots section problems 1, 2, 7, and 9-28). You should **not use a calculator** (except for the Powers and Square Roots section problems 18, 19, and 22-25).

You will be asked to **turn in your answers during orientation** in August. You will receive a grade equivalent to a quiz, with credit given based on the number of problems **attempted with work shown**. You do not have to have figured out how to get the correct answer as long as the work you did do is written down and turned in. We will be reviewing most or all of these problems and concepts over the first few weeks of school.

Thanks for your time,

Jonathan Frank

ADDING AND SUBTRACTING FRACTIONS

To add or subtract two fractions with the same denominator, add or subtract the numerators. Write the result in simplest form.

EXAMPLES Add or subtract.

a. $\frac{1}{8} + \frac{3}{8}$

b. $\frac{9}{10} - \frac{3}{10}$

Solution

a. $\frac{1}{8} + \frac{3}{8} = \frac{1+3}{8} = \frac{4}{8} = \frac{1}{2}$

b. $\frac{9}{10} - \frac{3}{10} = \frac{9-3}{10} = \frac{6}{10} = \frac{3}{5}$

To add or subtract two fractions with different denominators, write equivalent fractions with a common denominator. Then add or subtract and write the result in simplest form.

EXAMPLES Add or subtract.

a. $\frac{4}{15} + \frac{2}{5}$

b. $\frac{3}{4} - \frac{1}{6}$

Solution

a. Write $\frac{2}{5}$ as $\frac{6}{15}$.

b. Write $\frac{3}{4}$ as $\frac{9}{12}$ and $\frac{1}{6}$ as $\frac{2}{12}$.

$$\frac{4}{15} + \frac{2}{5} = \frac{4}{15} + \frac{6}{15} = \frac{10}{15} = \frac{2}{3}$$

$$\frac{3}{4} - \frac{1}{6} = \frac{9}{12} - \frac{2}{12} = \frac{7}{12}$$

Practice

Add or subtract. Write the answer in simplest form.

1. $\frac{2}{7} + \frac{4}{7}$

2. $\frac{7}{12} - \frac{1}{12}$

3. $\frac{2}{3} - \frac{1}{3}$

4. $\frac{1}{6} + \frac{1}{6}$

5. $\frac{3}{4} - \frac{1}{4}$

6. $\frac{4}{5} + \frac{1}{5}$

7. $\frac{7}{20} + \frac{9}{20}$

8. $\frac{79}{100} - \frac{14}{100}$

9. $\frac{7}{8} - \frac{1}{4}$

10. $\frac{2}{3} + \frac{1}{6}$

11. $\frac{3}{4} + \frac{1}{2}$

12. $\frac{5}{12} - \frac{1}{3}$

13. $\frac{5}{7} - \frac{2}{5}$

14. $\frac{3}{16} + \frac{5}{8}$

15. $\frac{3}{8} - \frac{1}{12}$

16. $\frac{2}{3} + \frac{1}{2}$

17. $\frac{5}{8} + \frac{3}{8}$

18. $\frac{24}{25} - \frac{3}{5}$

19. $\frac{5}{9} + \frac{14}{15}$

20. $\frac{9}{10} - \frac{4}{5}$

21. $\frac{2}{3} + \frac{1}{5}$

22. $\frac{3}{8} - \frac{7}{40}$

23. $\frac{17}{20} - \frac{1}{5}$

24. $\frac{7}{11} + \frac{1}{3}$

MULTIPLYING AND DIVIDING FRACTIONS

To multiply two fractions, multiply the numerators and multiply the denominators. Then write the result in simplest form.

EXAMPLES Multiply.

a. $\frac{3}{4} \times \frac{5}{6}$

b. $20 \times \frac{4}{5}$

Solution

a. $\frac{3}{4} \times \frac{5}{6} = \frac{3 \times 5}{4 \times 6} = \frac{15}{24} = \frac{5}{8}$

b. $\frac{20}{1} \times \frac{4}{5} = \frac{20 \times 4}{1 \times 5} = \frac{80}{5} = 16$

To divide by a fraction, multiply by its reciprocal and write the product in simplest form.

EXAMPLES Divide.

a. $\frac{1}{5} \div \frac{5}{8}$

b. $9 \div \frac{7}{10}$

Solution

a. $\frac{1}{5} \div \frac{5}{8} = \frac{1}{5} \times \frac{8}{5} = \frac{1 \times 8}{5 \times 5} = \frac{8}{25}$

b. $9 \div \frac{7}{10} = \frac{9}{1} \times \frac{10}{7} = \frac{9 \times 10}{1 \times 7} = \frac{90}{7} = 12\frac{6}{7}$

Practice

Multiply or divide. Write the answer in simplest form.

1. $\frac{1}{2} \times \frac{3}{4}$

2. $\frac{2}{3} \times \frac{3}{11}$

3. $20 \times \frac{1}{8}$

4. $65 \times \frac{2}{5}$

5. $\frac{5}{12} \times \frac{4}{9}$

6. $\frac{7}{8} \times \frac{1}{4}$

7. $16 \times \frac{15}{16}$

8. $10 \times \frac{1}{3}$

9. $\frac{1}{3} \div \frac{1}{2}$

10. $\frac{3}{4} \div \frac{5}{6}$

11. $9 \div \frac{1}{7}$

12. $14 \div \frac{1}{2}$

13. $\frac{7}{8} \div \frac{7}{2}$

14. $\frac{4}{5} \div \frac{1}{10}$

15. $100 \div \frac{7}{8}$

16. $40 \div \frac{3}{4}$

17. $\frac{2}{5} \div \frac{1}{5}$

18. $\frac{1}{4} \times \frac{7}{10}$

19. $\frac{22}{7} \times 49$

20. $\frac{8}{9} \div 4$

21. $\frac{8}{3} \times \frac{3}{8}$

22. $\frac{1}{5} \div \frac{2}{5}$

23. $\frac{3}{11} \div \frac{3}{11}$

24. $\frac{5}{7} \times \frac{2}{15}$

25. $12 \div \frac{3}{2}$

26. $\frac{7}{2} \times \frac{1}{14}$

27. $\frac{6}{5} \div \frac{3}{10}$

28. $4 \times \frac{3}{16}$

POWERS AND SQUARE ROOTS

An expression like 5^3 is called a **power**. The **exponent** 3 represents the number of times the **base** 5 is used as a factor: $5^3 = 5 \cdot 5 \cdot 5 = 125$.

EXAMPLES Evaluate.

a. 4^5

b. $(-10)^2$

Solution

a. $4^5 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 1024$

b. $(-10)^2 = (-10)(-10) = 100$

If $b^2 = a$, then b is a **square root** of a . Every positive number has two square roots, one positive and one negative. The two square roots of 16 are 4 and -4 because $4^2 = 16$ and $(-4)^2 = 16$. The radical symbol $\sqrt{\quad}$ indicates the *nonnegative* square root, so $\sqrt{16} = 4$.

EXAMPLES Find all square roots of the number.

a. 25

b. -81

Solution

a. Since $5^2 = 25$ and $(-5)^2 = 25$, the square roots are 5 and -5 .

b. Since -81 is negative, it has *no square roots*. There is no real number you can square to get -81 .

The square of an integer is a **perfect square**, so the square root of a perfect square is an integer.

Integer, (n)	1	2	3	4	5	6	7	8	9	10	11	12
Perfect square, (n^2)	1	4	9	16	25	36	49	64	81	100	121	144

You can approximate the square root of a positive number that is *not* a perfect square by using a calculator and rounding.

EXAMPLES Evaluate. Give the exact value if possible. Otherwise, approximate to the nearest tenth.

a. $\sqrt{49}$

b. $\sqrt{5}$

Solution

a. Since 49 is a perfect square with $7^2 = 49$, $\sqrt{49} = 7$.

b. Since 5 is not a perfect square, use a calculator and round: $\sqrt{5} \approx 2.2$.

A number or expression inside a radical symbol is called a **radicand**. The **simplest form of a radical expression** is an expression that has no perfect square factors other than 1 in the radicand, no fractions in the radicand, and no radicals in the denominator of a fraction.

You can use the following properties to simplify radical expressions.

Product Property of Radicals $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ where $a \geq 0$ and $b \geq 0$

Quotient Property of Radicals $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ where $a \geq 0$ and $b > 0$

EXAMPLES

Simplify.

a. $\sqrt{18}$

b. $\sqrt{\frac{9}{4}}$

c. $\frac{1}{\sqrt{2}}$

Solution

a. $\sqrt{18} = \sqrt{9 \cdot 2} = \sqrt{9} \cdot \sqrt{2} = 3 \cdot \sqrt{2} = 3\sqrt{2}$ Factor using perfect square factor.

b. $\sqrt{\frac{9}{4}} = \frac{\sqrt{9}}{\sqrt{4}} = \frac{3}{2}$ Use the quotient property and simplify.

c. $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ Write an equivalent fraction that has no radicals in the denominator.

Practice

Evaluate.

1. 8^2

2. $(-3)^2$

3. $(-1)^3$

4. 4^3

5. 2^5

6. 10^4

7. $(-9)^2$

8. 6^3

Find all square roots of the number or write *no real square roots*.

9. 100

10. -25

11. 1

12. 49

13. -9

14. 0

15. -36

16. 64

Evaluate. Give the exact value if possible. If not, approximate to the nearest tenth.

17. $\sqrt{100}$

18. $\sqrt{2}$

19. $\sqrt{15}$

20. $\sqrt{144}$

21. $\sqrt{4}$

22. $\sqrt{87}$

23. $\sqrt{11}$

24. $\sqrt{32}$

25. $\sqrt{45}$

26. $\sqrt{36}$

27. $\sqrt{0}$

28. $\sqrt{81}$

Simplify.

29. $\sqrt{28}$

30. $\sqrt{27}$

31. $\sqrt{50}$

32. $\sqrt{48}$

33. $\sqrt{\frac{5}{16}}$

34. $\sqrt{\frac{36}{49}}$

35. $\sqrt{\frac{1}{9}}$

36. $\sqrt{\frac{3}{25}}$

37. $\frac{1}{\sqrt{3}}$

38. $\frac{10}{\sqrt{2}}$

39. $\frac{5}{\sqrt{2}}$

40. $\frac{3}{\sqrt{3}}$

USING FORMULAS

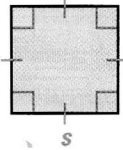
A **formula** is an algebraic equation that relates two or more real-life quantities. Here are some formulas for the perimeter P , area A , and circumference C of some common figures.

Square

side length s

$$P = 4s$$

$$A = s^2$$

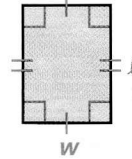


Rectangle

length ℓ and width w

$$P = 2\ell + 2w$$

$$A = \ell w$$

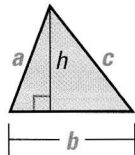


Triangle

side lengths a , b , and c
base b and height h

$$P = a + b + c$$

$$A = \frac{1}{2}bh$$

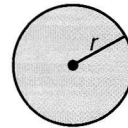


Circle

radius r

$$C = 2\pi r$$

$$A = \pi r^2$$



Pi (π) is the ratio of a circle's circumference to its diameter.

EXAMPLE

Find the length of a rectangle with perimeter 20 centimeters and width 4 centimeters.

Solution

$$P = 2\ell + 2w$$

Write the appropriate formula.

$$20 = 2\ell + 2(4)$$

Substitute known values of the variables.

$$20 = 2\ell + 8$$

Simplify.

$$12 = 2\ell$$

Subtract 8 from each side.

$$6 = \ell$$

Divide each side by 2.

ANSWER ▶ The length of the rectangle is 6 centimeters.

Practice

- The perimeter of a square is 24 meters. Find the side length.
- Find the area of a circle with radius 1.5 centimeters. (Use $\pi \approx 3.14$.)
- A triangle has a perimeter of 50 millimeters and two sides that measure 14 millimeters each. Find the length of the third side.
- Find the width of a rectangle with area 32 square feet and length 8 feet.
- The circumference of a circle is 8π inches. Find the radius.
- Find the side length of a square with area 121 square centimeters.
- Find the height of a triangle with area 18 square meters and base 4 meters.
- A square has an area of 49 square units. Find the perimeter.

Mercy High School

Plane Geometry

Summer Work Answer Key

Adding and Subtracting Fractions

- | | | |
|---------------------|----------------------|------------------|
| 1. $\frac{6}{7}$ | 2. $\frac{1}{2}$ | 3. $\frac{1}{3}$ |
| 4. $\frac{1}{3}$ | 5. $\frac{1}{2}$ | 6. 1 |
| 7. $\frac{4}{5}$ | 8. $\frac{13}{20}$ | 9. $\frac{5}{8}$ |
| 10. $\frac{5}{6}$ | 11. $1\frac{1}{4}$ | |
| 12. $\frac{1}{12}$ | 13. $\frac{11}{35}$ | |
| 14. $\frac{13}{16}$ | 15. $\frac{7}{24}$ | |
| 16. $1\frac{1}{6}$ | 17. 1 | |
| 18. $\frac{9}{25}$ | 19. $1\frac{22}{45}$ | |
| 20. $\frac{1}{10}$ | 21. $\frac{13}{15}$ | |
| 22. $\frac{1}{5}$ | 23. $\frac{13}{20}$ | |
| | 24. $\frac{32}{33}$ | |

Multiplying and Dividing Fractions

- | | | |
|--------------------|-------------------|-------------------|
| 1. $\frac{3}{8}$ | 2. $\frac{2}{11}$ | 3. $2\frac{1}{2}$ |
| 4. 26 | 5. $\frac{5}{27}$ | 6. $\frac{7}{32}$ |
| 7. 15 | 8. $3\frac{1}{3}$ | 9. $\frac{2}{3}$ |
| 10. $\frac{9}{10}$ | 11. 63 | |
| 12. 28 | 13. $\frac{1}{4}$ | |

- | | |
|---------------------|----------------------|
| 14. 8 | 15. $114\frac{2}{7}$ |
| 16. $53\frac{1}{3}$ | 17. 2 |
| 18. $\frac{7}{40}$ | 19. 154 |
| 20. $\frac{2}{9}$ | 21. 1 |
| 22. $\frac{1}{2}$ | 23. 1 |
| 24. $\frac{2}{21}$ | 25. 8 |
| 26. $\frac{1}{4}$ | 27. 4 |
| 28. $\frac{3}{4}$ | |

Powers and Square Roots

Powers

- | | | |
|---------------|--------|-------|
| 1. 64 | 2. 9 | 3. -1 |
| 4. 64 | 5. 32 | |
| 6. 10,000 | 7. 81 | |
| | 8. 216 | |
| 9. 10 and -10 | | |

Positive and Negative Roots

- | |
|--------------------------|
| 10. no real square roots |
| 11. 1 and -1 |
| 12. 7 and -7 |
| 13. no real square roots |
| 14. 0 |
| 15. no real square roots |
| 16. 8 and -8 |

17-28: In Geometry we assume roots are positive unless stated otherwise. Use a calculator for these problems:

- | | |
|---------|---------|
| 17. 10 | 18. 1.4 |
| 19. 3.9 | 20. 12 |
| 21. 2 | 22. 9.3 |
| 23. 3.3 | 24. 5.7 |
| 25. 6.7 | 26. 6 |
| 27. 0 | 28. 9 |

29-40: Do not use a calculator for these problems:

- | | |
|---------------------------|--------------------------|
| 29. $2\sqrt{7}$ | 30. $3\sqrt{3}$ |
| 31. $5\sqrt{2}$ | 32. $4\sqrt{3}$ |
| 33. $\frac{\sqrt{5}}{4}$ | 34. $\frac{6}{7}$ |
| 35. $\frac{1}{3}$ | 36. $\frac{\sqrt{3}}{5}$ |
| 37. $\frac{\sqrt{3}}{3}$ | 38. $5\sqrt{2}$ |
| 39. $\frac{5\sqrt{2}}{2}$ | 40. $\sqrt{3}$ |

Using Formulas

- | | |
|----------|-------------------------|
| 1. 6 m | 2. 7.07 cm ² |
| 3. 22 mm | 4. 4 ft |
| 5. 4 in | 6. 11 cm |
| 7. 9 m | 8. 28 u |